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# Optimizing Loan Portfolios

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## Introduction

The work of Nobel Laureate Harry Markowitz in the early 1950s triggered a revolution in the profession of investment management.[1] The concepts of *efficient portfolio* and *efficient frontier* served as catalysts for the development of modern finance. The basic idea is that investors should rationally choose portfolios that offer the highest return for the least amount of risk. This is done by diversifying among several securities, reducing risk as compared to the combined risks of the constituents. Each portfolio can be classified along the axes of risk and return. Any portfolio that has a minimal amount of risk for a given amount of return is called *efficient*, and the line that connects these portfolios in a risk-return graph is called the *efficient frontier.* Although Markowitz focused on securities, his novel theories have found their way into many industries and environments. Optimal credit portfolio selection in banking and the optimization of energy distribution are just two examples.

In 1993, Terri Gollinger and John Morgan, at the time working with Mellon Bank in Pittsburgh, published the pioneering article “Calculation of an Efficient Frontier for a Commercial Loan Portfolio” in the *Journal of Portfolio Management*.[2] This article takes Markowitz’s portfolio theory to the banking sector and to the allocation and optimization of loan portfolios in particular. In their approach, the industry sectors take the place of securities in the Markowitz

[1] Markowitz, H., Portfolio Selection, *The Journal of Finance*, March 1952, pp. 77-91.

[2] Gollinger, T.L. and J.B. Morgan, Efficient Frontier for a Commercial Loan Portfolio, *The Journal of Portfolio Management*, Winter 1993, pp. 39-46.

model, and the industry Zeta-scores are used as a proxy for risk. This Zeta-score represents the likelihood of a company going bankrupt in the next two years. Just as an investor searches for an optimal combination of risk and return in creating a portfolio of securities, a bank wants to extend loans to those industries that minimize risk for a given level of return.

In this paper we will try to emulate a simplified version of the Gollinger-Morgan approach. We deviate slightly by taking the standard deviation (σ) of the probability of default (PD) as an indicator for risk. In the first section of this paper, we use the common mathematical approach to calculate the risk of a diversified loan portfolio in Microsoft Excel. Then, we employ Oracle Crystal Ball to perform the same calculation, demonstrating that similar results can be obtained from a probabilistic approach. Subsequently, we attempt to improve the model by introducing skewed lognormal distributions. Thus, we will be steering away from the implicit assumption that financial assets should be modeled using a Gaussian normal distribution. Finally, we will optimize the loan portfolio for risk and return, and locate the efficient frontier.

## The Loan Portfolio

Consider a bank that is active in five industries: real estate, information and communications technology (ICT), fast-moving consumer goods (FMCG), leisure, and financial institutions (FI). Each of these industries has different characteristics in terms of riskiness and profitability. Each also has a different probability of default—that is, the chance that an obligor will not meet his or her financial obligations to the bank. This probability of default is equal to the average expected loss on a loan and is defined by the average (PD) and the standard deviation (σ PD, see Table 1).

Depending on the sector, the bank charges its clients different rates (expressed here as margins). The margin is the interest rate levied on top of the bank's funding rate, which is assumed to be EURIBOR, the Euro Interbank Offered Rate. The average expected loss on a loan is subtracted from the margin as if it were an expense. By so doing, the bank arrives at its return on assets (ROA). For simplicity's sake, we assume that the loss given default is 100 percent—which means it is basically ignored in our calculations. Finally, the bank's decision on the relative sizes of the sectors in the total loan portfolio is shown in the Portfolio Share column.

TABLE 1. LOAN PORTFOLIO OF A BANK

| PORTFOLIO | PD | MARGIN | ROA | Σ PD | PORTFOLIO SHARE |
|---|---|---|---|---|---|
| REAL ESTATE | 1.40% | 2.30% | 0.90% | 3.32% | 20% |
| ICT | 2.60% | 4.00% | 1.40% | 6.00% | 20% |
| FMCG | 2.20% | 3.00% | 0.80% | 4.00% | 20% |
| LEISURE | 2.40% | 2.90% | 0.50% | 3.61% | 20% |
| FI | 2.20% | 3.30% | 1.10% | 5.48% | 20% |

## Correlation

Calculating the average return of the loan portfolio is a straightforward operation: you simply multiply each industry's ROA with its weight and then sum the results. This yields an average portfolio return of 0.94 percent. The risk associated with the portfolio, as expressed in the standard deviation of the PD, is not as simple to calculate. For that, one has to take into account the correlations between the different industries. These correlations present a major risk driver for a bank. As such, any bank will seek a portfolio of loans that are uncorrelated, or even negatively correlated, to benefit most from diversification effects. However, such a portfolio is hard to come by. In practice, most industry sectors show a strong positive correlation. Table 2 displays the correlations of the probabilities of defaults among the five industries.

**TABLE 2. CORRELATIONS OF PROBABILITIES OF DEFAULT IN THE LOAN PORTFOLIO**

| PD CORRELATION MATRIX | REAL ESTATE | ICT | FMCG | LEISURE | FI |
|---|---|---|---|---|---|
| REAL ESTATE | 1 | 85% | 80% | 75% | 90% |
| ICT | 85% | 1 | 80% | 75% | 95% |
| FMCG | 80% | 80% | 1 | 70% | 80% |
| LEISURE | 75% | 75% | 70% | 1 | 75% |
| FI | 90% | 95% | 80% | 75% | 1 |

## Portfolio Risk

We are now ready to calculate the standard deviation of the complete loan portfolio. For this, we use matrix multiplication. Besides the correlation matrix above, we need the weighted σ-matrix, which lists the product of the standard deviation and the relative weight of the portfolio on the diagonal axis. Note that the cells' values beyond the diagonal are all zero.

**TABLE 3. MATRIX WITH WEIGHTED SIGMAS ON THE DIAGONAL**

| WEIGHTED Σ-MATRIX | REAL ESTATE | ICT | FMCG | LEISURE | FI |
|---|---|---|---|---|---|
| REAL ESTATE | 0.66% | 0.00% | 0.00% | 0.00% | 0.00% |
| ICT | 0.00% | 1.20% | 0.00% | 0.00% | 0.00% |
| FMCG | 0.00% | 0.00% | 0.80% | 0.00% | 0.00% |
| LEISURE | 0.00% | 0.00% | 0.00% | 0.72% | 0.00% |
| FI | 0.00% | 0.00% | 0.00% | 0.00% | **1.10%** |

By multiplying the correlation and σ-matrices according to the formula below, we arrive at the covariance matrix. Intuitively, the covariance matrix expands the concept of correlation to a multidimensional environment.

*Covariance Matrix = Weighted σ-matrix * Correlation matrix * Weighted σ-matrix*

In Excel, this calculation can be performed using a nested MMULT function. This Excel array function helps us to implement a matrix multiplication.

| J | K | L | M | N | O |
|---|---|---|---|---|---|
| σ | Real estate | ICT | FMCG | Leisure | FI |
| Real estate | 0.66% | 0.00% | 0.00% | 0.00% | 0.00% |
| ICT | 0.00% | 1.20% | 0.00% | 0.00% | 0.00% |
| FMCG | 0.00% | 0.00% | 0.80% | 0.00% | 0.00% |
| Leisure | 0.00% | 0.00% | 0.00% | 0.72% | 0.00% |
| FI | 0.00% | 0.00% | 0.00% | 0.00% | 1.10% |
| | | | | | |
| Correlation | Real estate | ICT | FMCG | Leisure | FI |
| Real estate | 1 | 85% | 80% | 75% | 90% |
| ICT | 85% | 1 | 80% | 75% | 95% |
| FMCG | 80% | 80% | 1 | 70% | 80% |
| Leisure | 75% | 75% | 70% | 1 | 75% |
| FI | 90% | 95% | 80% | 75% | 1 |
| | | | | | |
| Covar | Real estate | ICT | FMCG | Leisure | FI |
| Real estate | =MMULT(MMULT(K2:O6;K9:O13);K2:O6) | | | | 0.007% |
| ICT | 0.007% | 0.014% | 0.008% | 0.006% | 0.012% |
| FMCG | 0.004% | 0.008% | 0.006% | 0.004% | 0.007% |
| Leisure | 0.004% | 0.006% | 0.004% | 0.005% | 0.006% |
| FI | 0.007% | 0.012% | 0.007% | 0.006% | 0.012% |

Figure 1. The covariance matrix is equal to the σ-matrix *times* the correlation matrix *times* the σ-matrix.

The sum of the cells in this covariance matrix generates the variance of the portfolio, as shown in Table 4.

The variance of the portfolio (σ^2 Portfolio) amounts to 0.17 percent. To arrive at the standard deviation, we only need to take the square root of the variance: 4.15 percent. Thus, we have calculated the riskiness of the bank's loan portfolio.

**TABLE 4. COVARIANCE MATRIX WITH ITS SUM UNDER Σ^2 PORTFOLIO**

| COVAR MATRIX | REAL ESTATE | ICT | FMCG | LEISURE | FI |
|---|---|---|---|---|---|
| REAL ESTATE | 0.004% | 0.0047 | 0.004% | 0.004% | 0.007% |
| ICT | 0.007% | 0.014% | 0.008% | 0.006% | 0.012% |
| FMCG | 0.004% | 0.008% | 0.006% | 0.004% | 0.007% |
| LEISURE | 0.004% | 0.006% | 0.004% | 0.005% | 0.006% |
| FI | 0.007% | 0.012% | 0.007% | 0.006% | 1.012% |
| Σ^2 PORTFOLIO | 0.17% | | | | |
| Σ PORTFOLIO | 4.15% | | | | |

## Using Oracle Crystal Ball

With the help of Oracle Crystal Ball, we can arrive at the same results in a more practical and intuitive way. The PDs can be modeled using a normal distribution defined by the mean—in our case the expected PD—and a standard deviation (σ PD).

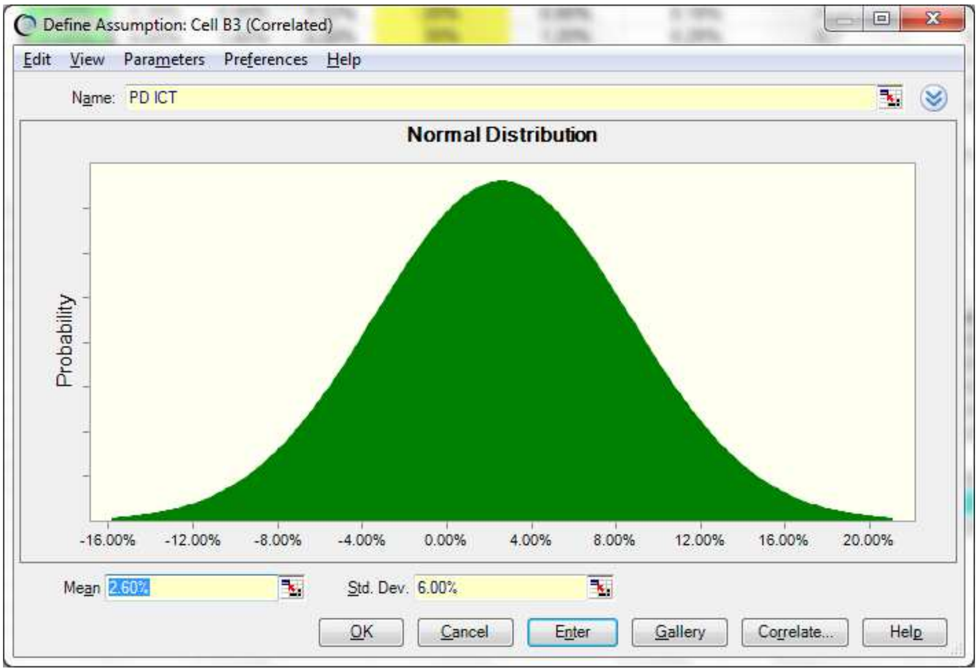


Figure 2. Modeling the PD of the ICT sector with a normal distribution.

We model all of the PDs in this way, while the other input parameters remain the same. The variable we are interested in is the return on the portfolio. It is defined as the output variable of the simulation and modeled as an Oracle Crystal Ball forecast. Note that Oracle Crystal Ball input parameters are in green and the output variable is in sky-blue.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | | PD | Margin | ROA | σ PD | Portfolio Share | ROA*Share |
| 2 | Real estate | 1.40% | 2.30% | 0.90% | 3.32% | 20% | 0.18% |
| 3 | ICT | 2.60% | 4.00% | 1.40% | 6.00% | 20% | 0.28% |
| 4 | FMCG | 2.20% | 3.00% | 0.80% | 4.00% | 20% | 0.16% |
| 5 | Leisure | 2.40% | 2.90% | 0.50% | 3.61% | 20% | 0.10% |
| 6 | FI | 2.20% | 3.30% | 1.10% | 5.48% | 20% | 0.22% |
| 7 | Totals | | | | | 100% | 0.94% |

Figure 3. Oracle Crystal Ball portfolio model.

Oracle Crystal Ball includes the correlations between the PDs as defined in the correlation matrix. In the figure below we see the Oracle Crystal Ball correlation definition for the PD Real Estate input variable.

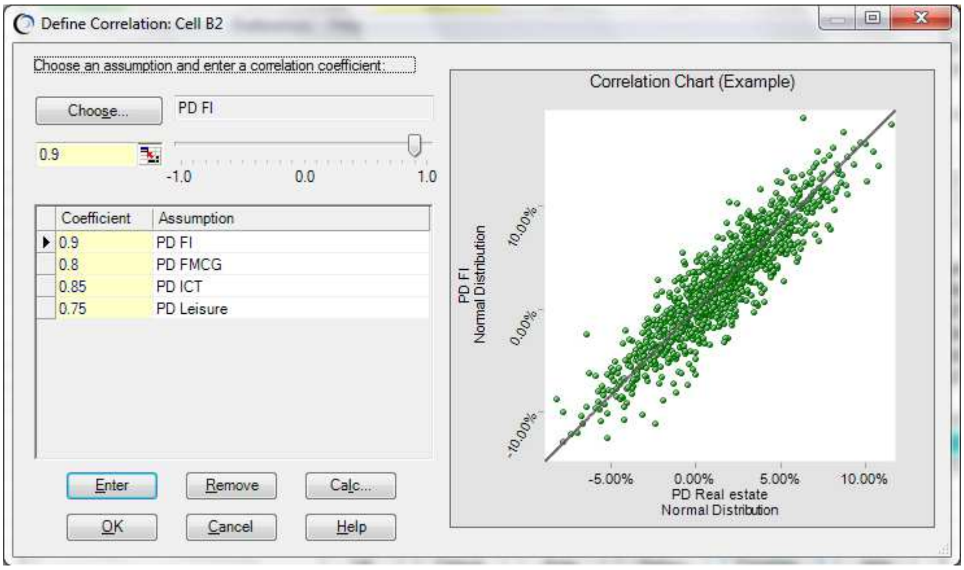


Figure 4. The PD correlation of the real estate industry.

The Monte-Carlo simulation will generate random values and the forecast field will follow suit. With 10,000 trials, Oracle Crystal Ball finds the same variance we calculated in Excel previously. Also the standard deviation is, barring some rounding errors, identical.

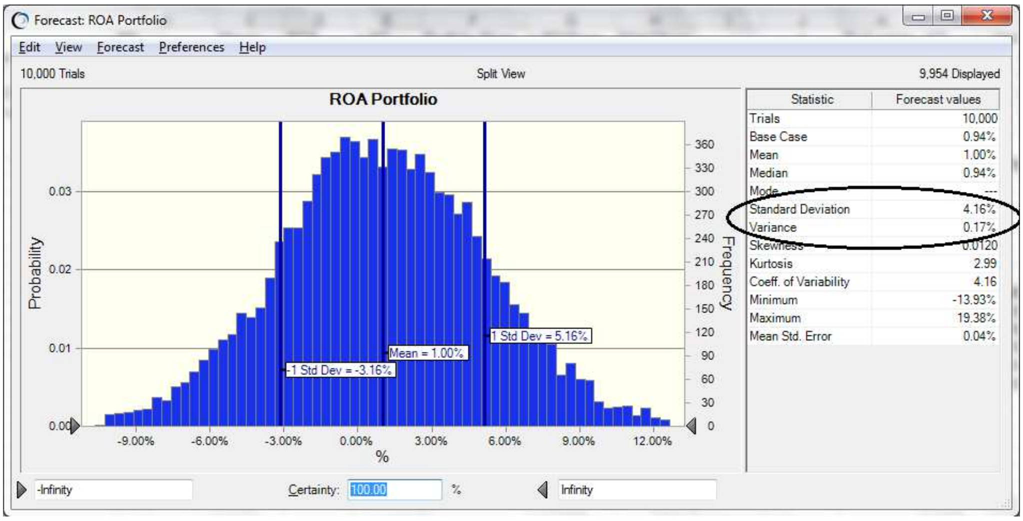


Figure 5. Histogram of the return on the loan portfolio.

## The Effects of Diversification

It would be interesting to see the benefits of diversification. We could easily run the simulation again, only this time with all of the correlation coefficients at 100 percent. This would be identical to having all obligors in a single industry sector—that is, holding a completely undiversified portfolio.

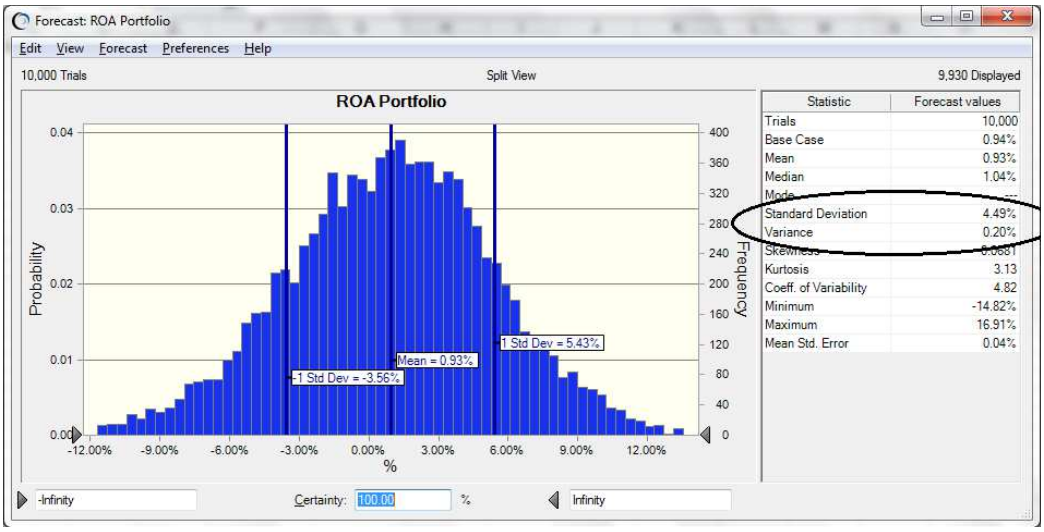


Figure 6. Portfolio with 100 percent industry correlation.

The results show that the variance of the portfolio increases from 0.17 percent to 0.20 percent, and the standard deviation is up by 33 basis points. This difference quantifies the portfolio benefit. This may not seem much, but to a bank this will make a difference. Thus, even in a highly correlated industry portfolio, gains can be made from diversification.

## A Sprinkle of Realism

The Markowitz portfolio theory assumes normal distributions for the behavior of the securities studied. Reams of paper have been devoted to the question of whether price movements of financial assets take a random walk that can be modeled with a bell curve. Indeed, ever since Benoît Mandelbrot's conducted studies on the misbehavior of markets, we have known that price movements of stocks are more erratic than most models account for.[3] For the purpose of this paper, we will not follow Mandelbrot all the way, but we will inquire further the shortcomings of using the normal distribution in the model explained above.

To start with, normal distributions hardly assign any probability to events beyond four or five standard deviations from the mean. Although the tail of the distribution will never touch the horizontal axis, in practice, the probability at five standard deviations equals zero. Thus, using the normal distribution leads us to ignore extreme events that occur more frequently in financial markets than these models suggest. A second problem pertains to the fact that the normal distribution is symmetric, while in

[3] Hudson, Richard L.; Mandelbrot, Benoît B. (2004); "The (Mis)Behaviour of Markets: A Fractal View of Risk, Ruin, and Reward."

reality, phenomena in financial markets tend to be highly skewed. In our case, it is clear that the PDs cannot be negative, whereas on the right-hand side the PD can go up to a 100 percent. Therefore, this PD is highly skewed to the right.

Oracle Crystal Ball features multiple asymmetric distributions; we will pick the lognormal distribution.

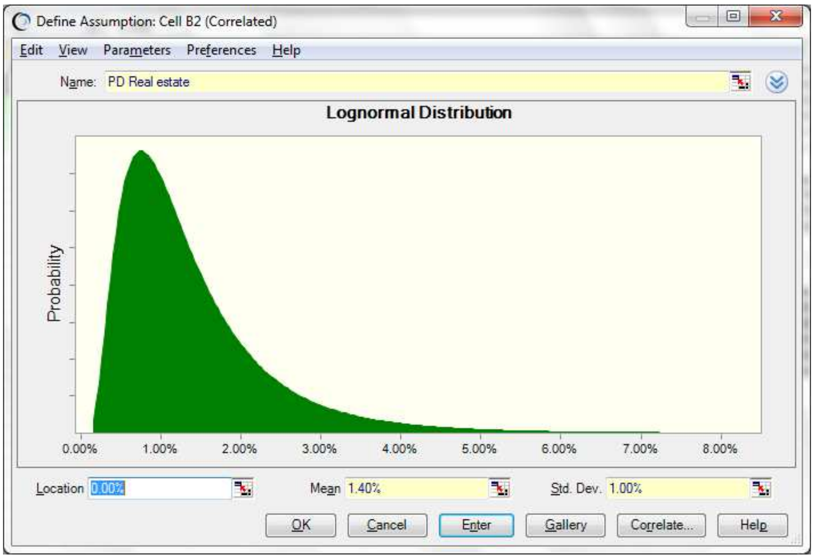


Figure 7. PD modeled with a lognormal distribution.

For one thing, the lognormal distribution has a location, which limits the domain—in our case, to positive values. Secondly, the distribution is highly skewed to the right, allowing for tail events to happen. The fat, long tail of the distribution gives room for modeling extreme events occurring many standard deviations away from the mean. One could say that this distribution provides a more realistic assessment of the risks involved. Running the simulation now produces a completely different picture.

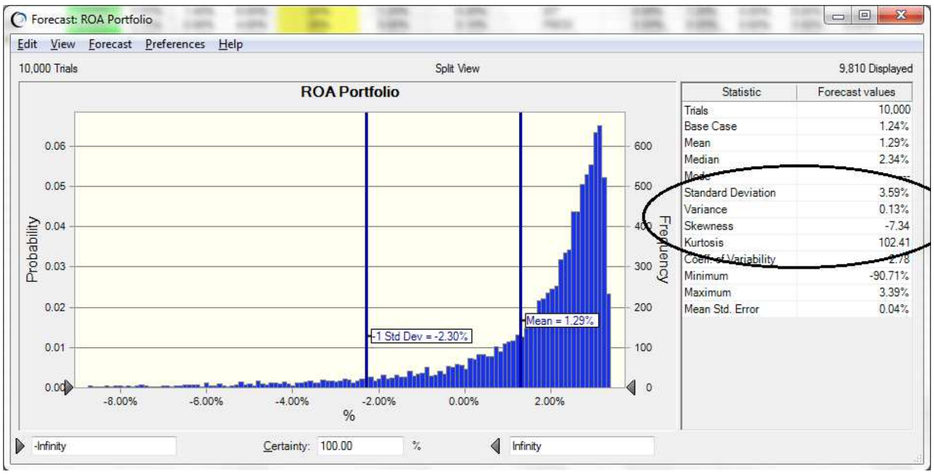


Figure 8. Loan portfolio returns using lognormal distributions.

The first thing we notice is the fat, long tail to the left, indicating relatively high chances for serious losses. We have to bear in mind that we have modeled the PD with a lognormal distribution that carries a fat long tail to the right. Since the PD is subtracted from the interest margin charged by the bank, high PD values now result in a negative performance of the loan portfolio, creating a fat tail to the left.

Secondly, we observe a small reduction in uncertainty as compared to the Gaussian approach, with lower values for variance and standard deviation. This indicates that, on average, the observations are less scattered. However, in terms of risk, we should look beyond these two summary statistics. The graph already shows that serious losses present a far from uncommon danger, making a complete wipeout of the portfolio a realistic scenario. To assess risk, it is a good idea to look at two other statistics: *skewness* and *kurtosis*.

*Skewness* provides the degree of asymmetry of the forecast chart, with zero being a perfectly symmetrical graph and large positive or negative values pointing toward a high probability of tail events. *Kurtosis* is the degree of "peakedness" of a graph, where a value of three represents the shape of a normal distribution and values above three indicate peaky graphs with high tail probability. Often, skewness and kurtosis are harbingers of relatively high chances of extreme risk. Hence, we see that the introduction of lognormal distributions has significant effects on our analysis. Oracle Crystal Ball offers a useful environment for this type of modeling, allowing us to attain a better and more realistic assessment of risk.

## Finding the Optimal Portfolio

What is the best way to allocate the lending capacity of the bank across various industries? Basically, this means finding the industry weights that result in the most efficient solutions. So far, we have kept the weights of the industry sectors constant by freezing them at 20 percent. Oracle Crystal Ball has the option to include decision variables in a model. For the decision parameters, Oracle Crystal Ball will propose the optimal values considering the objectives, requirements, and constraints defined. In our example, the objective is to optimize the ROA of the portfolio by deciding on the portfolio shares. Furthermore, we have put forward the requirement that the standard deviation of the portfolio should not exceed 4 percent. This limits the risk the bank is willing to take on. Solutions with a higher ROA, but an uncertainty exceeding this ceiling, will be discarded. In addition, we have defined the constraint that the weights should add up to 100 percent. Also, a minimum weight of 10 percent was defined, ensuring that the bank keeps a presence in all sectors.

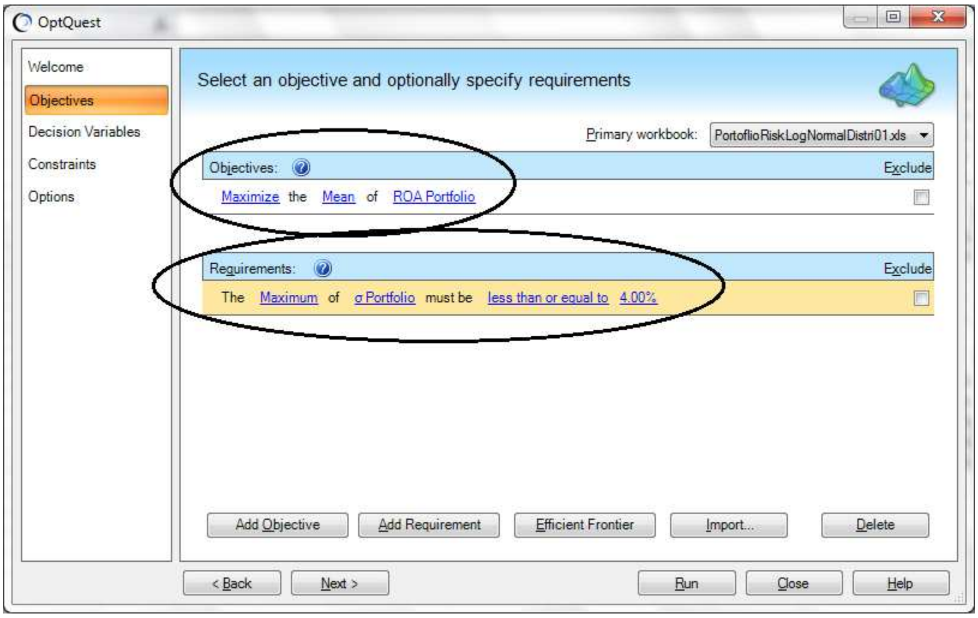


Figure 9. Defining objectives and requirements for optimization.

With these parameters in place, we can run the optimization simulation. The simulation returns the portfolio shares listed in the Figure 10..

The optimization exercise has provided a clear answer to the question of how to allocate the bank's lending capacity.

| | PD | Margin | ROA | σ PD | Portfolio Share | σ |
|---|---|---|---|---|---|---|
| Real estate | 1.40% | 2.30% | 0.90% | 3.32% | 10% | |
| ICT | 2.60% | 4.00% | 1.40% | 6.00% | 10% | |
| FMCG | 2.20% | 3.10% | 0.90% | 4.00% | 10% | |
| Leisure | 2.40% | 3.50% | 1.10% | 3.61% | 45% | |
| FI | 2.20% | 4.10% | 1.90% | 5.48% | 25% | |
| | | | | | 100% | R |

Figure 10. Optimized weights for the industry sectors.

The optimal solution found is valid for a risk ceiling that, in our case, was set at 4 percent. Markowitz demonstrated in his portfolio theory that any portfolio is defined along the axes of risk and return. This implies that a different maximum standard deviation will result in a different optimum for the portfolio allocation. By varying the risk ceiling and running the optimization simulation multiple times, a graph will emerge that Markowitz baptized the efficient frontier.

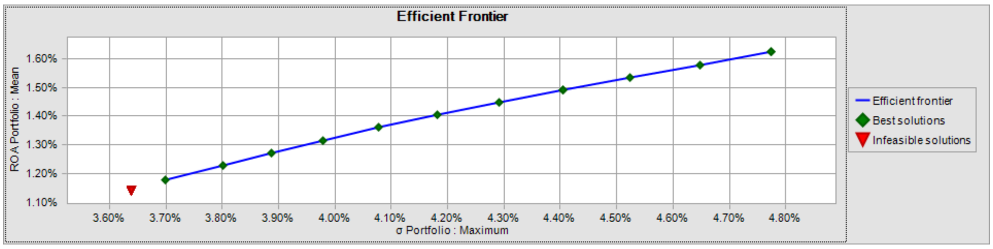


Figure 11. Efficient frontier with the risk level on the x-axis and ROA on the y-axis.

From the efficient frontier, we learn that the optimal solution regarding the composition of the loan portfolio depends on the bank's risk appetite. As conventional business wisdom confirms, risk and return go hand in hand.

## Conclusion

In this white paper, we combined Markowitz's portfolio theory with Gollinger and Morgan's loan portfolio idea to construct an Excel model that calculates the portfolio risk. Departing from the traditional approach, we have demonstrated how similar results can be achieved by introducing an Oracle Crystal Ball Monte-Carlo simulation. Conventional portfolio theory assumes the use of normal distributions. We have discussed some of the shortcomings of this approach and shown how, using lognormal distributions, this model can be improved upon, yielding new insights on uncertainty. Then, we took the modeling a step further by having the simulation determine the optimal settings for the allocation of a loan portfolio. Finally, we were able to generate an efficient frontier graph that describes the risk-return trade-off for bank credit.

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